

## Terminal Sliding Mode Control for Lane Changing of Vehicle With Estimation of Sideslip Velocity

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**Abstract:** In this paper, the terminal sliding mode control for lane changing of intelligent vehicle was studied. With positive and negative trapezoidal constraint of lateral acceleration, the desired yaw angle and yaw rate of vehicle was generated by the lane changing trajectory. Based on the estimation of sideslip velocity, by applying terminal sliding mode technology, the yaw-rate tracking control law for lane changing was designed. Based on the Lyapunov method, the stability of the control system is analyzed. Simulation results show that the control system is asymptotically stable.

**Keywords** - intelligent vehicle; lane changing; terminal sliding mode, estimation of sideslip velocity

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### I. Introduction

Lane change is the process of controlling a vehicle from one lane to another along a desired lane-changing trajectory[1]. In order to realize the automatic lane change, the lane change trajectory should be planned according to the vehicle running state and road information, and then the control law for lane change should be designed to track the desired trajectory. There are many methods for the planning of lane change trajectory, such as the vehicle longitudinal position and the desired lateral position satisfying the constraints of arc, sine or polynomial function, the desired lateral acceleration satisfying the constraints of positive and negative trapezoid, etc.[2].

On the design of vehicle lane-changing controller, Ref.[3] uses the optimal control theory to design the vehicle lane change control law, Ref.[4] uses the fuzzy logic to research on lane changing control of vehicle. Due to the uncertainty of vehicle parameters, Ref.[5] studied the adaptive control for lane change. In order to improve the convergence speed of the control system, Ref.[6] uses the terminal sliding mode method to study the vehicle lane change.

We know that the change of sideslip speed directly affects the control effect of vehicle lane change, however the above-mentioned literature has not studied the estimation of sideslip speed. In this paper, the uncertainty of vehicle sideslip velocity is considered, based on the lateral dynamical model of vehicle, the terminal sliding mode control method is used to study the control law for lane change of vehicle and the estimation formula for sideslip velocity was deduced by using Lyapunov function method.

### II. Expected lane changing trajectory

During the lane changing process, it is assumed that the lateral acceleration rate  $J_d(t)$  of the vehicle is expected as shown in Fig.1.

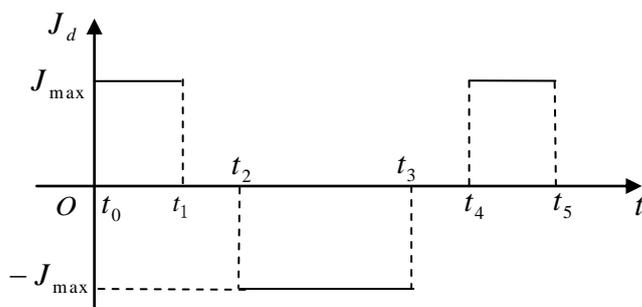


Fig.1 Desired lateral acceleration rate

Where  $t_0$  is the start time of lane change,  $t_5$  is the end time of lane change, and  $t_1 - t_0 = t_5 - t_4$ ,  $t_2 - t_1 = t_4 - t_3$ ,  $t_3 - t_2 = 2(t_1 - t_0)$ ,  $J_{\max}$  is the expected maximum of lateral acceleration rate. Let  $t_1 - t_0 = \Delta_1$ ,  $t_2 - t_1 = \Delta_2$ , then  $t_3 - t_2 = 2\Delta_1$ ,  $t_4 - t_3 = \Delta_2$ ,  $t_5 - t_4 = \Delta_1$ , the time required to change lanes is  $4\Delta_1 + 2\Delta_2$ . From Fig.2, the desired lateral displacement  $y_d(t)$  is obtained as follows.

$$y_d(t) = \begin{cases} 0, & t < 0 \\ (1/6)J_{\max}t^3, & 0 \leq t \leq t_1 \\ (J_{\max}/6)[\Delta_1^3 + 3\Delta_1^2(t - \Delta_1) + 3\Delta_1(t - \Delta_1)^2], & t_1 < t \leq t_2 \\ (J_{\max}/6)\{3\Delta_1(\Delta_1 + \Delta_2)(2t - 2\Delta_1 - \Delta_2) - (t - 2\Delta_1 - \Delta_2)^3\}, & t_2 < t \leq t_3 \\ (J_{\max}\Delta_1/6)\{2\Delta_1^2 - 6\Delta_1\Delta_2 - 3\Delta_2^2 + 3(\Delta_1 + 2\Delta_2)t - 3(t - 3\Delta_1 - \Delta_2)^2\}, & t_3 < t \leq t_4 \\ (J_{\max}/6)[12\Delta_1^3 + 18\Delta_1^2\Delta_2 + 6\Delta_1\Delta_2^2 + (t - 4\Delta_1 - 2\Delta_2)^3], & t_4 < t \leq t_5 \\ J_{\max}(2\Delta_1^3 + 3\Delta_1^2\Delta_2 + \Delta_1\Delta_2^2), & t > t_5 \end{cases} \quad (1)$$

### III. Control law design

The longitudinal axis direction should change along the tangential direction of the desired lane change trajectory, thus, the desired yaw angle and the yaw angular velocity of the vehicle are obtained as follows.

$$\psi_d(t) = \frac{\dot{y}_d(t)}{v_x}, \quad \dot{\psi}_d(t) = \frac{\ddot{y}_d(t)}{v_x} \quad (2)$$

Where  $v_x$  is the vehicle speed.

According to the automobile theory [7], the dynamic equation of yaw rate can be expressed as

$$\ddot{\psi} = -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \dot{\psi} - \frac{2(C_f l_f - C_r l_r)}{I_z v_x} v_y + \frac{2C_f l_f}{I_z} \delta + d_w \quad (3)$$

Where  $\psi$  is the vehicle yaw angle,  $v_y$  is the vehicle lateral velocity,  $m$  is the vehicle quality,  $I_z$  is the moment of inertia of vehicle around vertical axis,  $l_f$  and  $l_r$  represent the distance from the center of mass to the front axis and the distance from the center of mass to the rear axis, respectively.  $C_f$  and  $C_r$  respectively denote the lateral stiffness of the front and rear tires,  $\delta$  denotes the steering angle of vehicle.  $d_w$  is the interference factor, Assume that it changes slowly.

Design switching function

$$s = q_1(\dot{\psi} - \dot{\psi}_d) + q_2\left(\int_{t_0}^t \dot{\psi}(t)dt - \psi_d\right) \quad (4)$$

Where,  $q_1$  and  $q_2$  are all greater than 0. Take derivative of equation (4), we have

$$\dot{s} = q_1(\ddot{\psi} - \ddot{\psi}_d) + q_2(\dot{\psi} - \dot{\psi}_d) \quad (5)$$

From (3), we get

$$\begin{aligned} \dot{s} = & (-q_1 \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} + q_2) \dot{\psi} - q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} v_y \\ & + \frac{2q_1 C_f l_f}{I_z} \delta + q_1 d_w - q_1 \ddot{\psi}_d - q_2 \dot{\psi}_d \end{aligned} \quad (6)$$

According to  $\dot{s} = 0$ , we get the equivalent control as

$$\delta_{equ} = \frac{I_z}{2q_1 C_f l_f} \left[ (q_1 \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} - q_2) \dot{\psi} + q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} v_y - q_1 d_w + q_1 \ddot{\psi}_d + q_2 \dot{\psi}_d \right] \quad (7)$$

Assuming that  $\hat{d}_w$  and  $\hat{v}_y$  are the estimated values of  $d_w$  and  $v_y$ , respectively., then equation (6) rewrite as

$$\begin{aligned} \dot{s} = & (-q_1 \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} + q_2) \dot{\psi} - q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \hat{v}_y \\ & + \frac{2q_1 C_f l_f}{I_z} \delta + q_1 \dot{\hat{d}}_w - q_1 \ddot{\psi}_d - q_2 \dot{\psi}_d \end{aligned}$$

Further, we rewrite equivalent control (7) as

$$\delta_{equ} = \frac{I_z}{2q_1 C_f l_f} [(q_1 \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} - q_2) \dot{\psi} + q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \hat{v}_y - q_1 \dot{\hat{d}}_w + q_1 \ddot{\psi}_d + q_2 \dot{\psi}_d] \quad (8)$$

Let the nonlinear control term as

$$\delta_N = -\frac{I_z}{2q_1 C_f l_f} [\rho s + \phi s^{k/l}] \quad (9)$$

Where  $\rho > 0$  and  $\phi > 0$ ,  $k$  and  $l$  are positive odd numbers, and  $l > k$ . Design control law as

$$\delta = \delta_{equ} + \delta_N \quad (10)$$

The adaptive laws of the control parameters  $d_w$  is as follows.

$$\dot{\hat{d}}_w = \gamma s \quad (11)$$

Where  $\gamma > 0$ , is the adaptive rate correction factors.

From automobile theory, without considering the influence of wind force and road slope, the dynamic equation of vehicle sideslip velocity is

$$\dot{v}_y = -\frac{2(C_f + C_r)}{m v_x} v_y - [v_x + \frac{2(C_f l_f - C_r l_r)}{m v_x}] \dot{\psi} + \frac{2C_f}{m} \delta \quad (12)$$

The adaptive laws of side slip speed is designed as follows.

$$\begin{aligned} \dot{\hat{v}}_y = & -\frac{2(C_f + C_r)}{m v_x} \hat{v}_y - [v_x + \frac{2(C_f l_f - C_r l_r)}{m v_x}] \dot{\psi} + \frac{2C_f}{m} \delta \\ & - \alpha \frac{2(C_f l_f - C_r l_r)}{I_z v_x} s + \beta \tilde{v}_y \end{aligned} \quad (13)$$

Where  $\tilde{v}_y = v_y - \hat{v}_y$ , is the estimation error.

From (12) and (13), the dynamic equation of estimation error is obtained as follows.

$$\dot{\tilde{v}}_y = (-\frac{2(C_f + C_r)}{m v_x} - \beta) \tilde{v}_y + \alpha \frac{2(C_f l_f - C_r l_r)}{I_z v_x} s \quad (14)$$

Where  $\alpha > 0$ , and  $\beta > 0$ , are the observed gain constants

#### IV. Stability analysis

From (6), we get

$$\begin{aligned} \dot{s} = & (-q_1 \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} + q_2) \dot{\psi} - q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \tilde{v}_y + \frac{2q_1 C_f l_f}{I_z} \delta \\ & + q_1 \tilde{d}_w - q_1 \ddot{\psi}_d - q_2 \dot{\psi}_d - q_1 \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \hat{v}_y + q_1 \dot{\hat{d}}_w \end{aligned} \quad (15)$$

Where  $\tilde{d}_w = d_w - \hat{d}_w$ . Take Lyapunov function

$$V = \frac{1}{2}s^2 + \frac{1}{2}q_1\gamma^{-1}\tilde{d}_w^2 + \frac{1}{2}q_1\alpha^{-1}\tilde{v}_y^2 \tag{16}$$

The derivative of (16) is obtained by connecting the estimation error dynamics (14) and the variation rate of switching function (15).

$$\begin{aligned} \dot{V} &= s\dot{s} + q_1\gamma^{-1}\tilde{d}_w\dot{\tilde{d}}_w + q_1\alpha^{-1}\tilde{v}_y\dot{\tilde{v}}_y \\ &= s\left[(-q_1\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} + q_2)\dot{\psi} - q_1\frac{2(C_f l_f - C_r l_r)}{I_z v_x}\tilde{v}_y + \frac{2q_1 C_f l_f}{I_z}\delta\right. \\ &\quad \left.+ q_1\tilde{d}_w - q_1\dot{\psi}_d - q_2\dot{\psi}_d - q_1\frac{2(C_f l_f - C_r l_r)}{I_z v_x}\hat{v}_y + q_1\hat{d}_w\right] \\ &\quad - q_1\gamma^{-1}\tilde{d}_w\dot{\tilde{d}}_w - q_1\alpha^{-1}\tilde{v}_y\left[\left(\frac{2(C_f + C_r)}{m v_x} + \beta\right)\tilde{v}_y - \alpha\frac{2(C_f l_f - C_r l_r)}{I_z v_x}s\right] \end{aligned}$$

Control law (10) is substituted into the above equation, and its itemized formula (8) and (9) are considered, according to the interference estimation formula (11), we have

$$\begin{aligned} \dot{V} &= -\rho s^2 - \phi s^{(l+k)/l} - q_1\frac{2(C_f l_f - C_r l_r)}{I_z v_x}s\tilde{v}_y + q_1s\tilde{d}_w - q_1\gamma^{-1}\tilde{d}_w\dot{\tilde{d}}_w \\ &\quad - q_1\alpha^{-1}\tilde{v}_y\left[\left(\frac{2(C_f + C_r)}{m v_x} + \beta\right)\tilde{v}_y - \alpha\frac{2(C_f l_f - C_r l_r)}{I_z v_x}s\right] \\ &= -\rho s^2 - \phi s^{(l+k)/l} - q_1\alpha^{-1}\left(\frac{2(C_f + C_r)}{m v_x} + \beta\right)\tilde{v}_y^2 \leq 0 \end{aligned} \tag{17}$$

Since, by the corollary of Barbalat's lemma [8], the sliding mode is asymptotically reachable and the estimation error is asymptotically stable.

When the system state reaches the sliding mode surface, from (4), we have

$$q_1(\dot{\psi} - \dot{\psi}_d) + q_2\left(\int_{t_0}^t \dot{\psi}(t)dt - \psi_d\right) = 0 \tag{18}$$

Let  $e = \int_{t_0}^t \dot{\psi}(t)dt - \psi_d$ , then  $q_1\dot{e} + q_2e = 0$ . Due to  $q_1$  and  $q_2$  are all greater than 0, so we get

$\lim_{t \rightarrow \infty} e(t) = 0$ , and  $\lim_{t \rightarrow \infty} \dot{e}(t) = 0$ . Namely,  $\dot{\psi} \rightarrow \dot{\psi}_d$ , as  $t \rightarrow \infty$ .

### V. Simulation results

Assuming that lane spacing  $d_w$  is 3m, the expected value of maximum of lateral acceleration rate  $J_{max}$  is 0.2g/s, the expected value of maximum of lateral acceleration  $a_{max}$  is 0.1g, and the gravitational acceleration  $g$  is 10m/s<sup>2</sup>.

Assuming that initial value of sideslip displacement, sideslip velocity, yaw angular velocity and yaw angle are all 0; the vehicle weight is 2000kg, the moment of inertia is 3150 kgm<sup>2</sup>, the cornering stiffness of front and rear tires are 70 and 80 kN/rad, respectively, the distance from the center of mass to the front wheel is 1.33m, the distance from the center of mass to the rear wheel is 1.26m, and the longitudinal speed of the vehicle is 15m/s. The control law adopts formula (10), the adaptive laws of side slip speed is formula (13), and the initial estimation error is 0.1m/s.

Fig.2 and Fig.3 are the simulation results for lane changing tracking control, where the solid line represents the actual value, and the dotted line represents the expected value, respectively. Fig.2 shows the tracking performance of lateral velocity and position to the expected values when changing lane. Fig.3 reflects the changes of the vehicle yaw angle and yaw rate, it can be seen that the actual values of vehicle yaw angle and yaw rate have ideal tracking performance to the expected values.

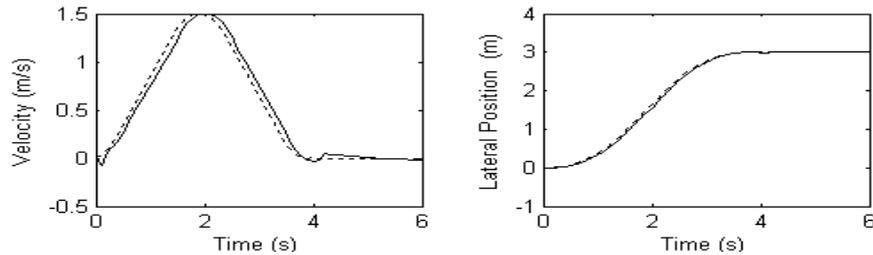


Fig.2 Simulation results of lateral velocity and position

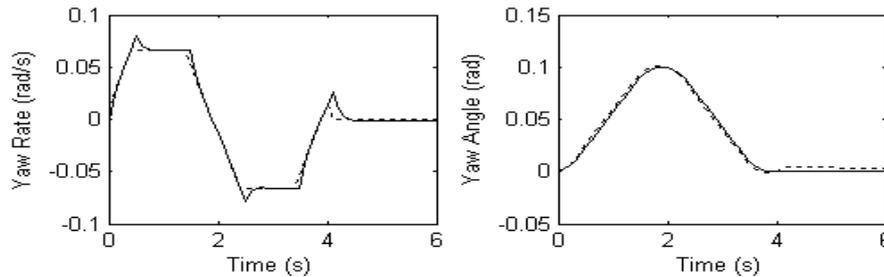


Fig.3 Simulation results of yaw angle and yaw rate

### Conclusion

In this paper, the uncertainty of vehicle sideslip velocity is considered. Based on the lateral dynamical model of vehicle, the estimation formula for sideslip velocity was deduced by using Lyapunov function method.

The terminal sliding mode control method is adopted to study the control law for lane change of vehicle, and the switching function is nonlinear. When the system state reaches the sliding surface, it will approach the equilibrium point in a limited time.

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